

The refractive index in the viscous quark-gluon plasma

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Under the framework of the viscous chromohydrodynamics, the gluon self-energy is derived for the quark-gluon plasma with shear viscosity. The viscous electric permittivity and magnetic permeability are evaluated from the gluon self-energy, through which the refraction index is investigated. The numerical analysis indicates that the refractive index becomes negative in some frequency range. The start point for that frequency range is around the electric permittivity pole, and the magnetic permeability pole determines the end point. As the increase of η/s , the frequency range for the negative refraction becomes wider.

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I. INTRODUCTION

Quantum chromodynamics (QCD) predicts that deconfined phase transition will take place at high temperature and/or high density, as a result the nuclear matter will undergo a transition to quark-gluon plasma (QGP). One main goal for Relativistic Heavy Ion Collider (RHIC) and Large Hadronic Collider (LHC) is to seek this new state of matter. There are two striking findings at RHIC. One is that the deconfined hot medium behaviors as a nearly perfect fluid with a small viscosity[1–4]. Several groups have applied viscous hydrodynamics to simulate the evolution of the produced matter in heavy ion collisions. The simulations successfully fit the observables at RHIC, such as the elliptic flow, the particle spectra, etc [5–13]. The other is the strong jet quenching, which is believed to be a potential signal for the QGP formation[14]. It should be stressed that the first LHC results also strongly support similar conclusions as seen at RHIC[15, 16].

At the very early stage of the relativistic heavy ion collisions, named glasma stage [17], and the late stage of the evolution process in the near T_c region in the so-called magnetic scenario for the QGP [18, 19], there are color-electric flux tubes which contain strong color-electric fields in them. Therefore, the color electromagnetic properties may play an important role in the evolution of hot and dense matter produced in heavy ion collisions. In addition, color electromagnetic properties could reflect the response of QGP to external color current, so the study of them may be helpful for understanding the nature of QGP. However, to the best of our knowledge, the study on the color electromagnetic properties of viscous QGP is scarce in literature. It makes sense to study how the viscosity affects them and how the viscous elec-

tromagnetic properties affect the evolution of produced matter in heavy ion collisions.

Refraction index is one important electromagnetic property which reflects the propagation of light in the medium. It can be determined in terms of the electric permittivity $\varepsilon(\omega, k)$ and magnetic permeability $\mu_M(\omega, k)$. In 1968, Veselago proposed in theory that the refraction index might be negative in some special material[20]. That kind of medium is in nature consistent with the one proposed by Mandelstam in which the electromagnetic phase velocity propagates antiparallel to the energy flow[21]. But no any natural material shows such special properties. Around 2000, by manipulating the array of small and closely spaced elements, scientists have constructed the negative refraction material in laboratory[23, 24], since then, the study on the negative refraction has attracted intensive interest. Recently, Amariti et al. have studied the refraction index of the strong coupled system with the string-inspired theory of AdS/CFT correspondence[25]. Then, some investigations have been carried out in strong coupled and correlation systems along that line[26–29]. It is argued that the negative refraction is a general phenomenon in some frequency range in charged fluid systems[29]. The probability for the existence of negative refraction in QGP is discussed as well in that literature[29]. Later, Juan Liu et al. extended the study of the refractive index of light to the weak coupled quark gluon system within the framework of the hard thermal loop perturbative theory[30].

In this paper, we will make a first step to study the refraction index of gluon in the viscous QGP with the viscous chromohydrodynamics. Gluon is the QCD counterpart of photon. In addition, jet quenching has been proposed as a potential signal for the QGP and become an active field in heavy ion collisions in last three decades, which is relevant to the parton propagation in the hot medium. So the study of gluon refraction in QGP may be helpful for the understanding of the nature of the QGP. According to the Refs.[31–33], viscosity will modify the distribution functions of the constituents of the QGP, thus it will affect the gluon self-energy through which the electric permittivity and magnetic permeability can

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be derived. Therefore, viscosity will have an impact on the refraction index.

It is argued that chromohydrodynamics can describe the polarization effect as the kinetic theory [34]. In a recent paper[35], some authors have extended the ideal chromohydrodynamics [36, 37] to the viscous one in terms of the QGP kinetic theory and the distribution function modified by the shear viscosity. Under that framework, the polarization tensor is derived and the color-electric permittivity in the viscous QGP is studied in details[35]. Based on the color-electric permittivity, the induced color charge distribution[38] and the corresponding wake potential[39] induced by the fast parton traveling through the viscous QGP have been investigated later.

In the present paper, by following the gluon polarization tension derived from the viscous chromohydrodynamics, we will derive the magnetic permeability and then study the refraction index in the QGP associated with shear viscosity. Our main results are as follows. Within the framework of the viscous chromohydrodynamics, the refraction index in the viscous QGP becomes negative in some frequency range. The start point of that frequency range is around the electric permittivity pole, while the magnetic permeability pole determines the end point. In addition, with the increase of η/s , the frequency range for the existence of the negative refraction becomes broadening.

The paper is organized as follows. In Section 2, we will briefly review the formulism of electromagnetic properties in medium. In Section 3, according to the polarization tension derived from the viscous chromohydrodynamics, we evaluated the refraction index and discussed the viscous effect on it. Section 4 is summary and remarks.

The natural units $k_B = \hbar = c = 1$, the metric $g_{\mu\nu} = (+, -, -, -)$ and the following notations $K = (\omega, \mathbf{k})$ are used in the paper.

II. THE ELECTROMAGNETIC PROPERTIES IN PLASMA

In order to describe the electric and magnetic properties in plasma covariantly, it is convenient to introduce a pair of four-vectors \tilde{E}^μ , \tilde{B}^μ in terms of the fluid four-velocity u^ν

$$\tilde{E}^\mu = u_\nu F^{\nu\mu}, \quad \tilde{B}^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\nu\lambda} u_\rho \quad (1)$$

and

$$F^{\mu\nu} = u^\mu \tilde{E}^\nu - \tilde{E}^\mu u^\nu + \epsilon^{\mu\nu\lambda\rho} \tilde{B}_\lambda u_\rho, \quad (2)$$

where the Greek index μ is not confused with the magnetic permeability μ_M . According to Eqs.(1) and (2),

one can obtain the Fourier-transformed free action

$$S_0 = -\frac{1}{2} \int \frac{d^4 K}{(2\pi)^4} \{ \tilde{E}^\mu(K) \tilde{E}_\mu(-K) - \tilde{B}^\mu(K) \tilde{B}_\mu(-K) \}. \quad (3)$$

Taking into account the interaction between the constituents of plasma, the correction to the action is

$$S_{int} = -\frac{1}{2} \int \frac{d^4 K}{(2\pi)^4} A^\mu(-K) \Pi_{\mu\nu}(K) A^\nu(K), \quad (4)$$

where $A^\mu(K)$ is vector boson field in momentum space, and $\Pi_{\mu\nu}(K)$ is polarization tensor which embodies the medium effects in plasma. In homogeneous and isotropic medium, the polarization tensor can be divided into longitudinal and transverse parts $\Pi_{\mu\nu}(K) = \Pi_L(K) P_{\mu\nu}^L(K) + \Pi_T(K) P_{\mu\nu}^T(K)$ with projector defined as $P_{00}^T = P_{0i}^T = P_{i0}^T = 0$, $P_{ij}^T = \delta^{ij} - \frac{k^i k^j}{k^2}$, $P_{\mu\nu}^L = \frac{k^\mu k^\nu}{K^2} - g^{\mu\nu} - P_{\mu\nu}^T$ [40, 41]. Thus, the effective action including medium effects is

$$S_{eff} = S_0 + S_{int}, \quad (5)$$

which also can be described as

$$S_{eff} = -\frac{1}{2} \int \frac{d^4 K}{(2\pi)^4} [\varepsilon \tilde{E}^\mu(K) \tilde{E}_\mu(-K) - \frac{1}{\mu_M} \tilde{B}^\mu(K) \tilde{B}_\mu(-K)]. \quad (6)$$

In (6), ε and μ_M represent the electric permittivity and magnetic permeability respectively which are the right quantities to describe the difference between the electric and magnetic properties of the vector field in the medium and those in the vacuum. According to Eqs.(3),(4) and (6), one can get the electric permittivity and magnetic permeability in plasma as following:

$$\varepsilon(\omega, k) = 1 - \frac{\Pi_L(\omega, k)}{K^2} \quad (7)$$

$$\frac{1}{\mu_M(\omega, k)} = 1 + \frac{K^2 \Pi_T(\omega, k) - \omega^2 \Pi_L(\omega, k)}{k^2 K^2} \quad (8)$$

We have briefly reviewed the electric and magnetic properties in homogeneous and isotropic plasma, the detailed derivation also can be found in Refs.[30, 42, 43]. In addition, the extension of the discussion to the anisotropic medium was also addressed in Ref.[30].

The refraction index is generally defined by the electric permittivity and magnetic permeability as $n^2 = \varepsilon(\omega, k) \mu_M(\omega, k)$ which is a square definition and not sensitive to the simultaneous change of signs of ε and μ_M . But it is proposed by Veselago that the simultaneous change of positive ε and μ_M to negative $-\varepsilon$ and $-\mu_M$ corresponds to the transformation of the refraction index from one branch $n = \sqrt{\varepsilon(\omega, k) \mu_M(\omega, k)}$ to the other $n = -\sqrt{\varepsilon(\omega, k) \mu_M(\omega, k)}$, ie, the turn from the general refraction index to the negative one. The physical nature of

the negative refraction is that the electromagnetic phase velocity propagates opposite to the energy flow. For the detailed discussion on the physics of negative refraction, please refer to the Refs.[20–22]. The criterion for the negative refraction is $\varepsilon < 0$ and $\mu < 0$ simultaneously for real electric permittivity and magnetic permeability medium.

If dissipation is taken into account, the situation is complicated. The electric permittivity and magnetic permeability are generally complex-valued functions of ω and k , such as $\varepsilon(\omega, k) = \varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k)$, $\mu_M(\omega, k) = \mu_r(\omega, k) + i\mu_i(\omega, k)$, so does the refraction index n . According to the phase velocity propagating antiparallel to the energy flow, some authors have derived the condition for negative refraction in dissipative medium and found it is not necessary for $\varepsilon_r < 0$ and $\mu_r < 0$ simultaneously[44]. Later, another simple, convenient and widely adopted condition has been derived as [45]

$$n_{eff} = \varepsilon_r |\mu_M| + \mu_r |\varepsilon| < 0, \quad (9)$$

where n_{eff} is called Depine-Lakhtakia index. $n_{eff} < 0$ implies $\text{Re } n < 0$, otherwise we will have a normal refraction index. In this paper, we will use criterion (9) to study if there exists the negative refraction in the viscous QGP.

III. THE REFRACTION INDEX IN THE VISCOUS QUARK-GLUON PLASMA

In this section, we will briefly review the derivation of the viscous chromohydrodynamics applicable to QGP with shear viscosity. Then, we will solve fluid equations to obtain the gluon polarization tensor. According to the gluon self-energy, the electric permittivity and magnetic permeability are determined, through which the refraction index will be investigated.

A. Kinetic theory

The kinetic equations for quarks, antiquarks and gluons are given by [46, 47]

$$p^\mu D_\mu Q(p, x) + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q(p, x)\} = C[Q, \bar{Q}, G], \quad (10)$$

$$p^\mu D_\mu \bar{Q}(p, x) - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}(p, x)\} = \bar{C}[Q, \bar{Q}, G], \quad (11)$$

$$p^\mu \mathcal{D}_\mu G(p, x) + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu G(p, x)\} = C_g[Q, \bar{Q}, G]. \quad (12)$$

$Q(p, x)$, $\bar{Q}(p, x)$ and $G(p, x)$ denote the distribution functions of quark, antiquark and gluon respectively. ∂_p^ν represents the four-momentum derivative and $\{\cdots, \cdots\}$ is the anticommutator. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$

represents the strength tensor in the fundamental representation, and $\mathcal{F}_{\mu\nu}$ is its counterpart in the adjoint representation. D_μ and \mathcal{D}_μ represent the covariant derivatives

$$D_\mu = \partial_\mu - ig[A_\mu(x), \cdots], \quad \mathcal{D}_\mu = \partial_\mu - ig[\mathcal{A}_\mu(x), \cdots]. \quad (13)$$

A_μ and \mathcal{A}_μ denote four-potentials in the fundamental and adjoint representations respectively,

$$A_\mu(x) = A_{\mu,a}(x)\tau^a, \quad \mathcal{A}_\mu(x) = T^a A_{\mu,a}(x), \quad (14)$$

where $a = 1, \dots, 8$; τ^a and T^a are the generators of group SU(3) in the corresponding representations; C , \bar{C} and C_g denote the collision terms.

The transport equations are supplemented by the Yang-Mills equation,

$$D_\mu F^{\mu\nu}(x) = j^\nu(x), \quad (15)$$

the color current $j^\nu(x)$ is given in the fundamental representation as

$$j^\nu(x) = -\frac{g}{2} \int_p p^\nu [Q(p, x) - \bar{Q}(p, x) - \frac{1}{3} \text{Tr}[Q(p, x) - \bar{Q}(p, x)] + 2\tau^a \text{Tr}[T^a G(p, x)]] \quad (16)$$

where $\int_p = \int \frac{d^4 p}{(2\pi)^3} 2\Theta(p_0) \delta(p^2)$.

Eqs.(10),(11),(12), (15) and (16) make up the fundamental equations of the kinetic theory for the quark-gluon plasma. In the linear approximation of QCD transport equation, by using the ideal, equilibrium distribution functions of constituents of the QGP, one can obtain the gluon self-energy[41, 46, 47]

$$\Pi_L(\omega, k) = m_D^2 (1 - \frac{\omega^2}{k^2}) [1 - \frac{\omega}{2k} \log[\frac{\omega + k}{\omega - k}]], \quad (17)$$

and

$$\Pi_T(\omega, k) = \frac{1}{2} m_D^2 [\frac{\omega^2}{k^2} + (1 - \frac{\omega^2}{k^2}) \frac{\omega}{2k} \log[\frac{\omega + k}{\omega - k}]], \quad (18)$$

where m_D is the Debye mass. (17) and (18) are consistent with those obtained in the hard thermal loop (HTL) approximation in diagrammatic methods at finite temperature field theory[40, 41, 43]. By combining with the HTL photon self-energy and Eqs.(7), (8) and (9), Juan Liu et al. have studied the refraction index of light in the QGP and found that it becomes negative in some frequency range.

B. Viscous chromohydrodynamics

In terms of Refs.[31–33], viscosity will modified the distribution function of the constituents of QGP system. If only shear viscosity is taken into account, the modified distribution function can be written as [33]

$$Q = Q_o + \delta Q = Q_o + \frac{c'}{2T^3} \frac{\eta}{s} Q_o (1 \pm Q_o) p^\mu p^\nu \langle \nabla_\mu u_\nu \rangle. \quad (19)$$

In Eq.(19), “+” is for boson, while “−” is for fermion. $c' = \pi^4/90\zeta(5)$ and $c' = 14\pi^4/1350\zeta(5)$ are for massless boson [12, 33] and massless fermion [48] respectively. $\langle \nabla_\mu u_\nu \rangle = \nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3}\Delta_{\mu\nu}\nabla_\gamma u^\gamma$, $\nabla_\mu = (g_{\mu\nu} - u_\mu u_\nu)\partial^\nu$, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$; η, s, T, Q_o represent the shear viscosity, the entropy density, the temperature of the system and the ideal distribution function of boson or fermion.

It is very difficult to evaluate the gluon self-energy with the QGP kinetic theory or finite temperature field theory associated with the distribution functions modified by shear viscosity Eq.(19). Fortunately, the fluid equations are rather simpler than the kinetic theory and usually used to study the plasma properties. By expanding the kinetic equations in momenta moments and truncating the expansion at the second level in terms of the ideal distribution function, chromohydrodynamics has been developed and been applied to study unstable modes of the QGP [36, 37].

By using the quark distribution function modified by shear viscosity Eq.(19) instead of the ideal one (Q_o) used in Refs. [36, 37] and doing the same momentum moments in terms of collisionless version of kinetic equation (10), we have extended the ideal chromohydrodynamic equations to the viscous ones [35]:

$$D_\mu n^\mu = 0, \quad D_\mu T^{\mu\nu} - \frac{g}{2}\{F_\mu^\nu, n^\mu(x)\} = 0 \quad (20)$$

with

$$n^\mu(x) = \int_p p^\mu Q(p, x), \quad T^{\mu\nu}(x) = \int_p p^\mu p^\nu Q(p, x). \quad (21)$$

The four-flow n^μ and energy momentum tensor $T^{\mu\nu}$ can be expressed in the form[35, 37]

$$n^\mu = n(x)u^\mu, \quad T^{\mu\nu} = \frac{1}{2}(\epsilon(x) + p(x))\{u^\mu, u^\nu\} - p(x)g^{\mu\nu} + \pi^{\mu\nu}, \quad (22)$$

where

$$\begin{aligned} \pi^{\mu\nu} &= \eta \langle \nabla^\mu u^\nu \rangle = \eta \{ (g^{\mu\rho} - u^\mu u^\rho) \partial_\rho u^\nu + (g^{\nu\rho} \\ &- u^\nu u^\rho) \partial_\rho u^\mu - \frac{2}{3} (g^{\mu\nu} - u^\mu u^\nu) \partial_\sigma u^\sigma \}. \end{aligned} \quad (23)$$

Because we only focus on the quark sector, the color current Eq.(16) reads

$$j^\mu(x) = -\frac{g}{2}(nu^\mu - \frac{1}{3}Tr[nu^\mu]). \quad (24)$$

Eqs.(20),(22) and (24) make up the basic set of equations of the viscous chromohydrodynamics. In those equations, n, ϵ and p represent the particle density, the energy density and pressure respectively. These quantities are $N_c \times N_c$ matrices in color space[37]. If $\eta = 0$, the distribution function remains the ideal form, $\pi^{\mu\nu}$ will be absent in (22) and the chromohydrodynamic equations will turn to the ideal ones[37].

C. Gluon self-energy

Linearizing the hydrodynamic quantities around the stationary, colorless and homogeneous state which is described by $\bar{n}, \bar{u}^\mu, \bar{p}$ and $\bar{\epsilon}$, as an example, the particle density is written as

$$n(x) = \bar{n} + \delta n(x). \quad (25)$$

The stationary and fluctuation quantities satisfy $\delta n \ll \bar{n}$ and $D_\mu \bar{n} = 0$. The corresponding parts of other hydrodynamic quantities have the similar properties. The color current $j^\mu(x)$ vanishes in the stationary state. All the fluctuations of the hydrodynamics quantities can contain both colorless and colored components, for example,

$$\delta n = \delta n_0 I_{\alpha\beta} + \frac{1}{2} \delta n_a \tau_{\alpha\beta}^a, \quad (26)$$

where $\alpha, \beta = 1, 2, 3$ are color indices and I is the identity matrix [37].

Substituting the linearized hydrodynamic quantities like Eq.(25) into Eq.(22) and their corresponding conservation equations (20) and projecting them on \bar{u}^μ and $(g^{\mu\nu} - \bar{u}^\mu \bar{u}^\nu)$, then, considering only the equations for colored parts of fluctuations and performing the Fourier transformation, one can obtain equations which can describe color phenomena in the viscous QGP[35]

$$\bar{n} k_\mu \delta u_a^\mu + k_\mu \delta n_a \bar{u}^\mu = 0, \quad (27)$$

$$\bar{u}^\mu k_\mu \delta \epsilon_a + (\bar{\epsilon} + \bar{p}) k_\mu \delta u_a^\mu = 0, \quad (28)$$

$$\begin{aligned} &(\bar{\epsilon} + \bar{p})(\bar{u} \cdot K) \delta u_a^\nu + (-k^\nu + \bar{u}^\nu (\bar{u} \cdot K)) \delta p_a \\ &+ \eta \{ (K^2 - (K \cdot \bar{u})) \delta u_a^\nu + (k^\mu k^\nu - k^\mu \bar{u}^\nu) \delta u_{\mu,a} \\ &+ \frac{2}{3} (\bar{u}^\nu (K \cdot \bar{u}) - k^\nu) k_\rho \delta u_a^\rho \} = i g \bar{n} \bar{u}_\mu F_a^{\mu\nu}(K). \end{aligned} \quad (29)$$

We introduce an EoS $\delta p_a = c_s^2 \delta \epsilon_a$ to complete the fluid equations, the explicit formulism for c_s will be introduced later. According to Eqs.(27),(28), (29) and the introduced EoS, we can obtain the colored fluctuations of hydrodynamic quantities $\delta n_a, \delta u_{\nu,a}$ and $\delta \epsilon_a$. Due to the color fluctuations of the hydrodynamic quantities, the color current fluctuation is given by

$$\delta j_a^\mu = -\frac{g}{2}(\bar{n} \delta u_a^\mu + \delta n_a \bar{u}^\mu - \frac{1}{3} Tr[\bar{n} \delta u_a^\mu + \delta n_a \bar{u}^\mu]). \quad (30)$$

Substituting into the solved δn_a and δu_a^μ , according to the relation between the current and the gauge field in the linear response theory $\delta j_a^\mu(K) = -\Pi_{ab}^{\mu\nu}(K) A_{\nu,b}(K)$, one can abstract the polarization tensor $\Pi_{ab}^{\mu\nu}(K)$ [35]

$$\begin{aligned} \Pi_{ab}^{\mu\nu}(\omega, k) &= -\delta_{ab} \{ \omega_p^2 \cdot \frac{1}{1 + D(K^2 - (K \cdot \bar{u})^2)} \cdot \frac{1}{(K \cdot \bar{u})^2} \\ &\cdot [(K \cdot \bar{u})(\bar{u}^\mu k^\nu + k^\mu \bar{u}^\nu) - K^2 \bar{u}^\mu \bar{u}^\nu \\ &- (K \cdot \bar{u})^2 g^{\mu\nu} + (B + E) \cdot [K^2 (K \cdot \bar{u})(\bar{u}^\mu k^\nu \\ &+ k^\mu \bar{u}^\nu) - k^\mu k^\nu (K \cdot \bar{u})^2 - K^4 \bar{u}^\mu \bar{u}^\nu]] \}, \end{aligned} \quad (31)$$

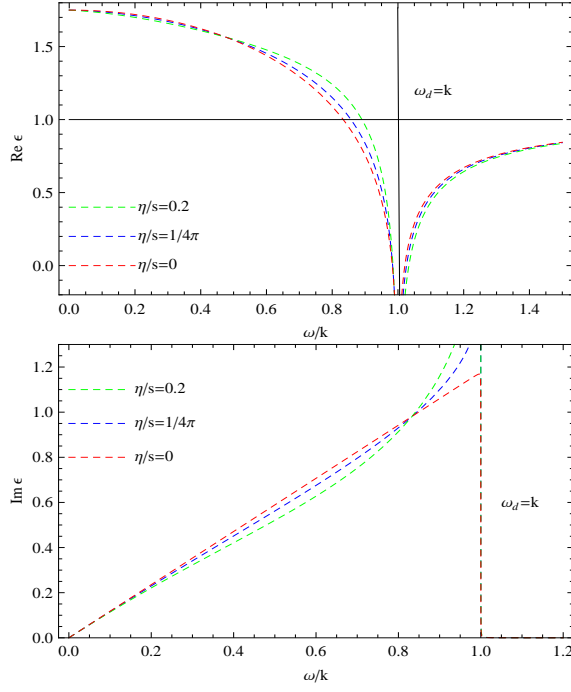


FIG. 1: (color online) The electric permittivity in the viscous QGP. Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue and green curves are for the cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively.

where ω_p is the plasma frequency and

$$\begin{aligned} B &= -\frac{c_s^2}{\omega^2 - c_s^2 k^2}, & D &= \frac{\eta}{sT\omega}, \\ E &= -\frac{\frac{\eta\omega}{sT}(1 + 4\frac{c_s^2 k^2}{\omega^2 - c_s^2 k^2})}{3\omega^2 - 3c_s^2 k^2 - 4\frac{\eta\omega k^2}{sT}}. \end{aligned} \quad (32)$$

It is easy to test that $\Pi_{ab}^{\mu\nu} = \Pi_{ab}^{\nu\mu}$ and $k_\mu \Pi_{ab}^{\mu\nu} = 0$. In the further considerations, we suppress the color indices a, b .

According to the projector, we can obtain the longitudinal and transverse gluon self-energy

$$\Pi_L(K) = \frac{K^2}{k^2} \Pi^{00}(K), \quad (33)$$

$$\Pi_T(K) = \frac{1}{2}(\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi^{ij}(K), \quad (34)$$

with $\hat{k}_i = k_i/k$.

We have briefly reviewed the determination of the gluon self-energy in the QGP associated with shear viscosity with the viscous chromohydrodynamic approach. For details please refer to Ref.[35]. Through the derivation, shear viscosity is encoded in the gluon self-energy. Combining with Eqs. (33), (34), (7), (8) and (9), we can study the refraction index of gluon in the viscous QGP.

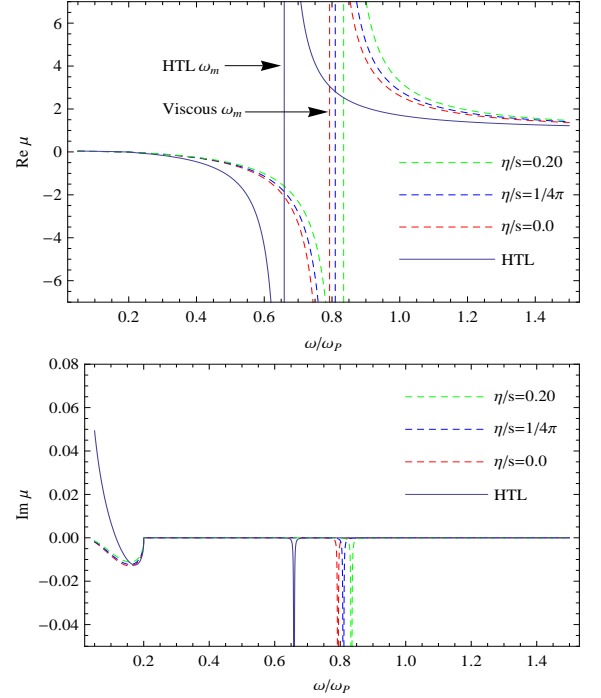


FIG. 2: (color online) The magnetic permeability in the QGP. Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue and green curves are for the cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively, while the solid blue curve is for the HTL case.

D. Numerical results

Before we do a further analysis on the refraction index, we should determine the sound speed c_s first. Mannarelli and Manuel have investigated collective unstable modes of QGP with the ideal chromohydrodynamic approach [37] as well as the kinetic theory [34]. They found that when one uses “the effective speed of sound” $c_s = \sqrt{\frac{1}{3(1 + \frac{1}{2y} \log \frac{1-y}{1+y})} + \frac{1}{y^2}}$ ($y = \frac{k}{\omega}$), the results in the chromohydrodynamic approach agree well with those in the kinetic theory in the same setting (see discussion in Appendix in Ref. [34]). In this paper, we also use the effective speed of sound.

In this paper, we can not determine shear viscosity coefficient itself in viscous chromohydrodynamics, but regard it as an input parameter to study the viscous effect on the refraction index of the QGP. A small value of the ratio for shear viscosity to entropy density $\eta/s \leq 0.2$ has been deduced from comparison of casual viscous hydrodynamic simulation results with the RHIC data[13], which is less than three times of the famous bound result $\eta/s = \frac{1}{4\pi}$ of the strongly couple conformal field theory determined by the AdS/CFT correspondence[49]. Numerical results of the refraction index are presented with those explicit values of η/s . In addition, in numerical analysis, such scales $k = 0.2\omega_p$ and $T = \omega_p$ are used to study the ω -dependent behavior of the refraction index.

In terms of the relation between Eq. (33) and (7), one can obtain the electric permittivity in soft momentum approximation [35]

$$\begin{aligned} \varepsilon(\omega, k) = & 1 + \frac{3\omega_P^2}{k^2} \left[1 - \frac{\omega}{2k} \left(\log \left| \frac{\omega + k}{\omega - k} \right| - i\pi\Theta(k^2 - \omega^2) \right) \right] \\ & - \frac{12\omega_P^2}{k^2} \frac{\eta\omega}{sT} \times \left\{ 1 - \frac{\omega}{k} \log \left| \frac{\omega + k}{\omega - k} \right| \right. \\ & + \frac{\omega^2}{4k^2} \left(\log \left| \frac{\omega + k}{\omega - k} \right| \right)^2 - \frac{\omega^2}{4k^2} \pi^2 \Theta(k^2 - \omega^2) \\ & \left. + i \left(\frac{\omega}{k} \pi - \frac{\omega^2}{2k^2} \pi \log \left| \frac{\omega + k}{\omega - k} \right| \right) \Theta(k^2 - \omega^2) \right\}, \quad (35) \end{aligned}$$

where Θ is the step function. It should be noted that the same result has been obtained in nonlinear viscous chromohydrodynamics in the soft limit of $\omega, k \ll T$ in a recent literature[50]. Here, we plot the real and imaginary parts of the electric permittivity in the viscous QGP in Fig.1. The main findings are as following. First, there is a frequency pole for the real part at $\omega_d = k$, which is just the inflexion of the imaginary part. Second, if such relation $m_D^2 = 3\omega_P^2$ is adopted[51], when $\eta/s = 0$, $\varepsilon(\omega, k)$ recovers the HTL result obtained by the kinetic theory or finite temperature field theory[40, 41, 43, 46, 47]. Third, the viscous corrections to both the real and imaginary parts of $\varepsilon(\omega, k)$ are small. For detailed discussion, please refer to Ref.[35].

According to the relation between Π_L , Π_T and $\mu_M(\omega, k)$, the magnetic permeability in the viscous QGP can be derived from Eqs. (33), (34) and (8)

$$\mu_M(\omega, k) = \frac{1}{1 + \frac{\omega_P^2}{k^2} \cdot \frac{1}{1 - \frac{\eta}{s} \cdot \frac{k^2}{T\omega}} + \frac{\omega^2}{k^2} \cdot (\varepsilon(\omega, k) - 1)}. \quad (36)$$

We present the real and imaginary parts of magnetic permeability of the viscous QGP in top and bottom panels respectively in Fig.2. For comparison, we also display the HTL results as well. The dashed red, blue and green curves are for the viscous cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively, while the blue solid curves are for the HTL results. Both the real and imaginary parts of magnetic permeability show up a frequency pole ω_m . Its position is around $0.65\omega_P$ for the HTL results, but around $0.8\omega_P$ for the viscous cases. In addition, it is easy to see that the frequency pole shifts to large frequency region with the increase of η/s .

From Eqs.(35), (36) and (9), we can determine the Depine-Lakhtakia index in the viscous QGP. We display its numerical results with different values of η/s in Fig.3 as well as the HTL result evaluated from Eqs.(17), (18), (7), (8) and (9). As shown in Fig.3, there is a quite large frequency range for $n_{eff} < 0$. In terms of the discussion in the section 2, in that frequency range refraction index becomes negative, ie. $\text{Re } n < 0$. With the increase of η/s , the frequency range for the negative refraction becomes wider. In addition, the frequency range for negative refraction in the viscous QGP is much wider than that of the HTL case.

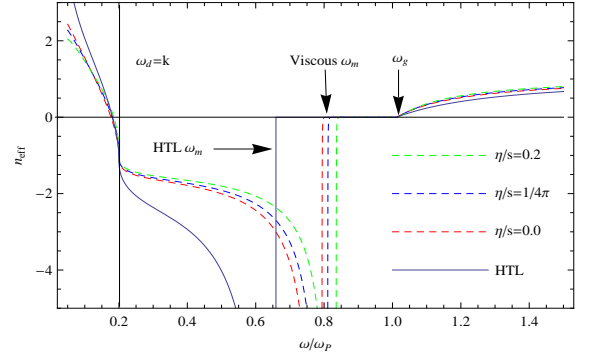


FIG. 3: (color online) The Depine-Lakhtakia index for the QGP. The dashed red, blue and green curves are for the cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively, while the solid blue curve is for the HTL case.

In Fig.3, one can see that, with the increase of the frequency, there is an inflexion for n_{eff} where n_{eff} changes from positive to negative value when the frequency ω is around ω_d , and n_{eff} is negative until $\omega = \omega_m$. The numerical analysis in Fig.1, Fig.2 and Fig.3 shows that the start point of the frequency region for negative refraction is around the electric permittivity ω_d , while the magnetic permeability pole ω_m determines the end point. Note that the electric permittivity poles in the viscous cases superpose each other, whose position coincides with that of the HTL result, as shown in Fig.1. Therefore, the start points for negative refraction show no appreciable distinction among all curves in Fig.3. From Fig.2, one can see that the magnetic permeability pole shifts to large frequency region with the increase of η/s , which leads to an enlargement of the frequency range for negative refraction. The magnetic permeability pole is around $0.65\omega_P$ for the HTL result, but around $0.8\omega_P$ for the viscous cases, which results in the fact that the frequency range for negative refraction in the viscous QGP is much wider than that of the HTL case.

From Fig.3, it is shown that both viscous curves and the HTL curve intersect one point at $\omega = \omega_g$. When $\omega > \omega_g$, $n_{eff} > 0$ for all curves, which implies a general refraction index. A frequency gap $\omega \in [\omega_m, \omega_g]$ is illustrated for $n_{eff} = 0$, in which $n^2 < 0$ as shown in Fig.4. It is argued that that result has not been reported in earlier literature[30]. The light does not propagate in that frequency gap, because the refraction index is pure imaginary and the electromagnetic wave is damped severely [30].

To obtain the refraction index n , one has to study the square root of $n^2 = \varepsilon\mu_M$. The complex value number $n^2 = \varepsilon\mu_M$ possesses two square roots,

$$n = \pm \sqrt{|n^2|} e^{i(\phi/2)}. \quad (37)$$

In (37), ϕ is the argument of $n^2 = \varepsilon\mu_M$ which can be expressed as

$$\phi = \phi_\varepsilon + \phi_\mu, \quad (38)$$

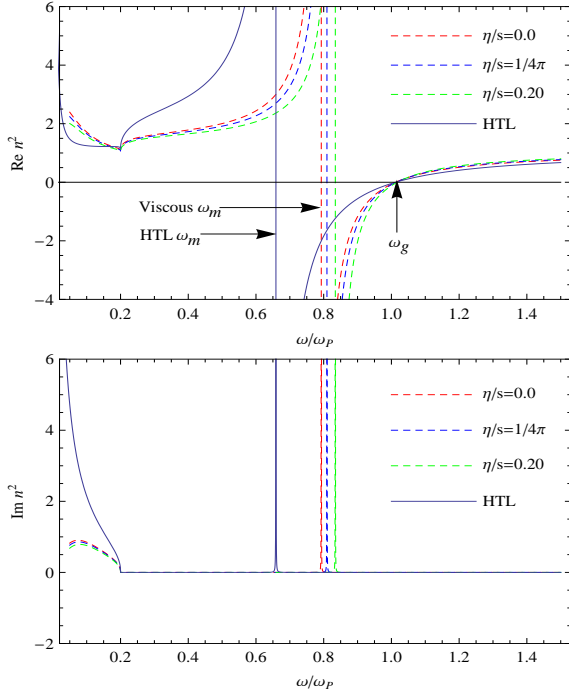


FIG. 4: (color online) The real and imaginary parts of n^2 in the QGP. Top panel: the real part. Bottom panel: imaginary part. The dashed red, blue and green curves are for the cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively, while the solid blue curve is for the HTL case.

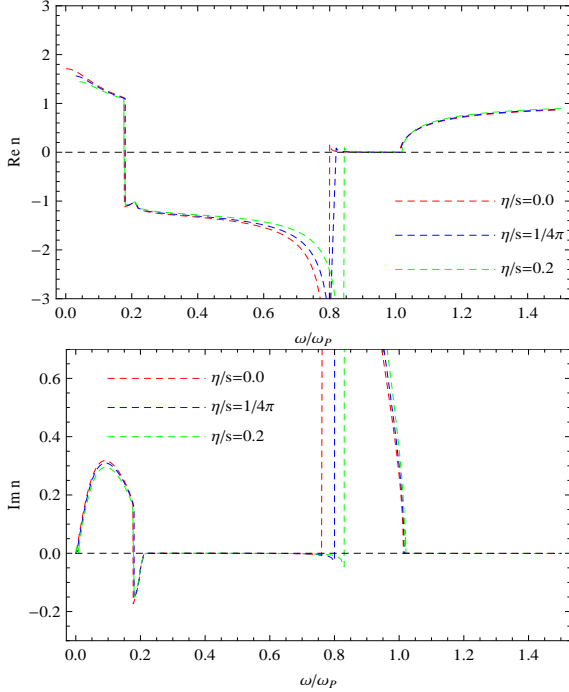


FIG. 5: (color online) The refractive index in the viscous QGP, Top panel: the real part. Bottom panel: imaginary part. The dashed red, blue and green curves are for the cases of $\eta/s = 0, 1/4\pi, 0.2$ respectively.

where $\phi_\varepsilon = \varepsilon_i/\varepsilon_r$ and $\phi_\mu = \mu_i/\mu_r$ are the arguments of ε and μ_M respectively. But which root in Eq.(37) is to be chosen? In terms of the criterion (9) that $n_{eff} < 0$ implying $\text{Re } n < 0$, otherwise $\text{Re } n > 0$, and the arguments of ε (35) and μ_M (36), we present the real and imaginary parts of n in the QGP with different η/s values in Fig.5. One can see that the properties of the real part of n is qualitatively consistent with the corresponding ones of n_{eff} . Only the frequency is around the magnetic permeability pole, the obvious difference demonstrate for the real and imaginary parts of n with the different η/s . Otherwise, the viscous correction to them is very trivial. The HTL results of n is also demonstrated in Ref.[30].

Due to simplicity and applicability to describe polarization effect, we have applied the viscous chromohydrodynamics, which is derived from the QGP kinetic theory and the viscosity-modified distribution function, to determine the polarization tensor and investigate the refraction index in the viscous QGP. It should be noted that some dynamical information will be lost during the derivation from the kinetic theory to the chromohydrodynamics[34, 37, 50]. However, in many cases the discrepancies between both approaches of the kinetic theory and the chromohydrodynamics can be alleviated by using effective parameters as inputs in the hydrodynamic formalisms[34, 50]. Nevertheless, such phenomenological model of the chromohydrodynamics could capture the some correct physics of the QGP[37, 50]. In view of the difficulty in investigating the viscous effect on the color electromagnetic properties of QGP in microscopic kinetic theory description, we expect that we could obtain some insight on the physics of the problem by applied the viscous chromohydrodynamics.

IV. SUMMARY

In this paper, within the framework of the viscous chromohydrodynamics, the gluon self-energy has been evaluated in the QGP associated with shear viscosity, through which the electric permittivity and magnetic permeability have been derived. Based on the viscous $\varepsilon(\omega, k)$ and $\mu_M(\omega, k)$, we have investigated the Depine-Lakhtakia index n_{eff} and the refraction index n in the viscous QGP. For comparison, we have also presented the corresponding HTL results. $n_{eff} < 0$ implies the negative real part of n , which signifies the negative refraction in the medium. The numerical analysis shows that i) the refraction index becomes negative in some frequency range; ii) the start point of that frequency range is around the pole of electric permittivity $\varepsilon(\omega, k)$, and the magnetic permeability pole determines the end point; iii) with the increase of the η/s , the frequency range for the negative refraction becomes broader. In addition, the frequency range for negative refraction in the viscous chromohydrodynamics is wider than that of the HTL perturbation theory. The numerical analysis also indicates that viscous properties of poles for ε and μ_M are responsible for

that difference.

The criterion (9) $n_{eff} < 0$ has been widely used to judge the existence of the negative refraction in a medium. It is interesting to study the interplay between the modes propagation ($\text{Re} n$) and dissipation ($\text{Im} n$) in the frequency region for negative refraction. For instance, in strongly coupled system with the framework of AdS/CFT correspondence, it was found that there exists strong dissipation in the frequency region for negative refraction, only around $\omega \rightarrow 0$, propagation may

dominate over dissipation[25]. How the shear viscosity impacts the mode propagation in QGP deserves further comprehensive studies.

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- [1] I. Arsene, et al., BRAHMS Collaboration, Nucl. Phys. A 757, 1 (2005), arXiv:nucl-ex/0410020.
 - [2] B. B. Back, et al., PHOBOS Collaboration, Nucl. Phys. A 757, 28 (2005), arXiv:nucl-ex/0410022.
 - [3] J. Adams, et al., STAR Collaboration, Nucl. Phys. A 757, 102 (2005), nucl-ex/0501009.
 - [4] K. Adcox, et al., PHENIX Collaboration, Nucl. Phys. A 757, 184 (2005), nucl-ex/0410003.
 - [5] R. Baier, P. Romatschke and U. A. Wiedemann, Phys. Rev. C 73, 064903 (2006), arXiv:hep-ph/0602249.
 - [6] P. Romatschke, Eur. Phys. J. C 52, 203 (2007), arXiv:nucl-th/0701032.
 - [7] M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008), arXiv:0804.4015[nucl-th].
 - [8] U. Heinz, Huichao Song and A. K. Chaudhuri, Phys. Rev. C 73, 034904 (2006), arXiv: nucl-th/0510014.
 - [9] Huichao Song and U. Heinz, Phys. Lett. B 658, 279 (2008), arXiv:0709.0742[nucl-th].
 - [10] Huichao Song and U. Heinz, Phys. Rev. C 77, 064901 (2008), arXiv:0712.3715 [nucl-th].
 - [11] Huichao Song and U. Heinz, Phys. Rev. C 78, 024902 (2008), arXiv:0805.1756 [nucl-th].
 - [12] K. Dusling and D. Teaney, Phys. Rev. C 77, 034905 (2008), arXiv:0710.5932 [nucl-th].
 - [13] Huichao Song, S. A. Bass, U. W. Heinz, T. Hirano and Chun Shen, Phys. Rev. Lett 106, 192301 (2011), arXiv: 1011.2783 [nucl-th].
 - [14] Xin-nian Wang, Nucl. Phys. A 750, 98 (2005), arXiv:nucl-th/0405017.
 - [15] K. Aamodt, et al., ALICE Collaboration, Phys. Rev. Lett 105, 252302 (2010), arXiv:1011.3914 [nucl-ex].
 - [16] G. Aad, et al., ATLAS Collaboration, Phys. Rev. Lett 105, 252303 (2010), arXiv:1011.6182 [hep-ex].
 - [17] T. Lappi and L. McLerran, Nucl. Phys. A 772, 200 (2006), arXiv:hep-ph/0602189.
 - [18] Jinfeng Liao and E. Shuryak, Phys. Rev. C 75, 054907 (2007), arXiv:hep-ph/0611131.
 - [19] Jinfeng Liao and E. Shuryak, Phys. Rev. Lett 101, 162302 (2008), arXiv:0804.0255 [hep-ph].
 - [20] V. G. Veselago, Sov. Phys. Usp 10, 509 (1968).
 - [21] V. M. Agranovich and Yu. N. Gartstein, Phys. Usp 49, 1029 (2006).
 - [22] S.A. Ramakrishna, Rep. Prog. Phys 68, 449 (2005)
 - [23] J. B. Pendry, Phys. Rev. Lett 85, 3966 (2000).
 - [24] D. R. Smith and N. Kroll, Phys. Rev. Lett 85, 2933 (2000).
 - [25] A. Amariti, D. Forcella, A. Mariotti and G. Policastro, JHEP 04(2011)036, arXiv:1006.5714 [hep-th].
 - [26] A. Amariti, et al, JHEP 10(2011)104, arXiv:1107.1242 [hep-th].
 - [27] Xian-hui Ge, K. Jo and Sang-jin Sin, JHEP 03 (2011) 104, arXiv:1012.2515 [hep-th].
 - [28] Xin Gao and Hong-bao Zhang, JHEP 08 (2010) 075, arXiv:1008.0720 [hep-th].
 - [29] A. Amariti, et al, arXiv:1107.1240 [hep-th].
 - [30] Juan Liu, M. J. Luo, Qun Wang and Hao-jie Xu, Phys. Rev. D 84, 125027 (2011), arXiv:1109.4083 [hep-ph].
 - [31] S. R. De Groot, W. A. Van Leeuwen and C. G. Van Weert, Relativistic Kinetic Theory, North-Holland, Amsterdam, 1980.
 - [32] P. Arnold, G. D. Moore and L. G. Yaffe, JHEP 11 (2000) 001, arXiv:hep-ph/0010177.
 - [33] D. Teaney, Phys. Rev. C 68, 034913 (2003), arXiv:nucl-th/0301099.
 - [34] M. Mannarelli and C. Manuel, Phys. Rev. D 77, 054018 (2008), arXiv:0707.3893 [hep-ph].
 - [35] Bing-feng Jiang and Jia-rong Li, Nucl. Phys. A 847, 268 (2010).
 - [36] C. Manuel and S. Mrówczyński, Phys. Rev. D 74, 105003 (2006), arXiv:hep-ph/0606276.
 - [37] M. Mannarelli and C. Manuel, Phys. Rev. D 76, 094007 (2007), arXiv:0705.1047 [hep-ph].
 - [38] Bing-feng Jiang and Jia-rong Li, J. Phys. G: Nucl. Part. Phys 39, 025007 (2012).
 - [39] Bing-feng Jiang and Jia-rong Li, Nucl. Phys. A 856, 121 (2011).
 - [40] J. I. Kapusta, Finite-Temperature Field Theory, Cambridge Univ. Press, Cambridge, 1989.
 - [41] M. Le Bellac, Thermal Field Theory, Cambridge Univ. Press, Cambridge, 1996.
 - [42] Jin Meng and Jia-rong Li, Nucl. Phys. A 730, 97 (2004).
 - [43] H. A. Weldon, Phys. Rev. D 26, 1394 (1982).
 - [44] M. W. McCall, A. Lakhtakia and W. S. Weiglhofer, Eur. J. Phys 23, 353 (2002), arXiv:physics/0204067 [physics.class-ph].
 - [45] R. A. Depine and A. Lakhtakia, Microw. Opt. Technol. Lett 41, 315 (2004), arXiv:physics/0311029 [physics.optics].
 - [46] H. -Th. Elze, U. Heinz, Phys. Rept 183, 81 (1989).
 - [47] S. Mrówczyński, Phys. Rev. D 39, 1940 (1989).
 - [48] K. Dusling and Shu Lin, Nucl. Phys. A 809, 246 (2008), arXiv:0803.1262 [nucl-th].
 - [49] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005), arXiv:hep-th/0405231.
 - [50] J. Peralta-Ramos and E. Calzetta, arXiv:1208.2715 [hep-ph].
 - [51] P. Chakraborty, M. G. Mustafa and M. H. Thoma, Phys. Rev. D 74, 094002 (2006), arXiv:hep-ph/0606316.